## Cambridge IGCSE ${ }^{\text {TM }}$



## CAMBRIDGE INTERNATIONAL MATHEMATICS

0607/52
Paper 5 Investigation (Core)
October/November 2021
1 hour 10 minutes
You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.


## INFORMATION

- The total mark for this paper is 36 .
- The number of marks for each question or part question is shown in brackets [ ].

This square dotty paper may be used for your diagrams.

Answer all the questions.

## INVESTIGATION

## CONNECTING DOTS

This investigation looks at the number of ways of connecting dots using straight lines.
This diagram shows 1 dot.
There is 1 row and 1 column.
This is a 1 by 1 diagram.
There are no connections to other dots.

This diagram shows 4 dots.
There are 2 rows and 2 columns.
This is a 2 by 2 diagram.
There are 6 ways to join 2 dots.
These are:

- 2 vertical connectors (solid lines)
- 2 horizontal connectors (solid lines)

- 1 up diagonal connector (dashed line)
- 1 down diagonal connector (dashed line).

1 (a) This is a 3 by 3 diagram.
The diagram shows:

- 6 horizontal connectors
- 4 up diagonal connectors.

Each connector joins 2 dots.


Complete the diagram by drawing the 6 vertical connectors and the 4 down diagonal connectors that join 2 dots.
(b) This is a 4 by 4 diagram.


On this 4 by 4 diagram,
(i) draw the horizontal connectors and the vertical connectors that join 2 dots,
(ii) draw the up diagonal connectors and the down diagonal connectors that join 2 dots.
(c) Complete the table for the numbers of connectors that join 2 dots.

Use part (b) and any patterns you notice.
You may use the square dotty paper on page 2 for diagrams.

|  |  | Numbers of connectors that join 2 dots |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Horizontal | Vertical | Up diagonal | Down diagonal | Total |  |  |
| 1 by 1 | 0 | 0 | 0 | 0 | 0 |  |  |
|  | 2 by 2 | 2 | 2 | 1 | 1 | 6 |  |
|  | 3 by 3 | 6 | 6 | 4 | 4 | 20 |  |
|  | 4 by 4 |  |  |  |  |  |  |
|  | 5 by 5 | 20 |  | 16 |  |  |  |
| 6 by 6 |  |  |  |  | 110 |  |  |

(d) In an $n$ by $n$ diagram there are $n$ rows and $n$ columns.
(i) Find an expression, in terms of $n$, for the number of up diagonal connectors that join 2 dots on an $n$ by $n$ diagram.
(ii) Find an expression, in terms of $n$, for the number of horizontal connectors that join 2 dots on an $n$ by $n$ diagram.
(e) Use your answers to part (d) to find the total number of connectors that join 2 dots on a 15 by 15 diagram.

2 This is a 3 by 3 diagram.
There are 8 ways to join $\mathbf{3}$ dots. These are:

- 3 vertical connectors
- 3 horizontal connectors
- 1 up diagonal connector
- 1 down diagonal connector.

(a) This is a 4 by 4 diagram.


Find the number of horizontal, vertical, up diagonal and down diagonal connectors that join 3 dots. Two horizontal connectors have been drawn for you.

Horizontal $\qquad$
Vertical $\qquad$
Up diagonal $\qquad$
Down diagonal $\qquad$
(b) Complete the table for the numbers of connectors that join 3 dots.

Use your answers to part (a) and any patterns you notice.
You may use the square dotty paper on page 2 for diagrams.

| Size of diagram ( $n$ by $n$ ) |  | Numbers of connectors that join 3 dots |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Horizontal | Vertical | Up diagonal | Down diagonal | Total |
|  | 2 by 2 | 0 | 0 | 0 | 0 | 0 |
|  | 3 by 3 | 3 | 3 | 1 | 1 | 8 |
|  | 4 by 4 |  |  |  |  |  |
|  | 5 by 5 | 15 |  |  |  |  |
|  | 6 by 6 |  |  |  |  | 80 |

(c) (i) This is an expression for the number of up diagonal connectors that join 3 dots on an $n$ by $n$ diagram.

$$
(n-2)^{2}
$$

Work out the number of up diagonal connectors that join 3 dots on a 20 by 20 diagram. diagram.

$$
n^{2}+a n
$$

Find the value of $a$ and write down the expression.

3 (a) Complete the table for the numbers of connectors that join 4 dots.

|  |  | Numbers of connectors that join 4 dots |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Horizontal | Vertical | Up diagonal | Down diagonal | Total |  |
|  | 3 by 3 | 0 | 0 | 0 | 0 | 0 |
| Size of <br> diagram <br> $(n$ by $n)$ | 4 by 4 |  |  |  |  | 10 |
|  | 5 by 5 | 10 |  |  |  |  |
|  | 6 by 6 | 18 | 18 | 9 | 9 | 54 |

(b) (i) Write down an expression, in terms of $n$, for the number of up diagonal connectors that join 4 dots on an $n$ by $n$ diagram.
(ii) Find an expression, in terms of $n$, for the number of horizontal connectors that join 4 dots on an $n$ by $n$ diagram.
(c) Show that the total number of connectors that join 4 dots on an $n$ by $n$ diagram is

$$
4 n^{2}-18 n+18
$$

(d) Find the size of the diagram which has a total of 180 connectors that join 4 dots.

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